# Math 103E Calculus with Analytic Geometry I

## Common Final Examination

## Sample 1

Name: _____

GCC ID # _____

Instructor: _____

**INSTRUCTIONS**

- To receive full credit, you must show all work and box final answers.
- If you need extra space, use the blank facing page.
- Simplify all answers completely and fully.
- Round all decimal answers to four places.
- No reference material of any kind is allowed.
- Keep your photo ID out on your desk during the entire exam.
- Only basic calculators and scientific calculators are allowed. No graphing calculators, computers, or any devices capable of storing alphabetic characters. No calculators embedded in other devices such as cell phones, watches, pagers, address books, etc. are allowed.

**Problem 1.** Use the limit definition of the derivative to find the derivative of $f(x) = 3/x^2$.

**Problem 2.** Find the derivative of the function $y = (\sinh x)^{\tan^{-1} x}$. Write the final answer in terms of $x$.

**Problem 3.** Consider the function $f(x) = \dfrac{e^{2x}}{1+e^{2x}}$.

(a) Find the derivative of $f$.

(b) Find the equation of the tangent line to $f$ at $x = 0$. Write the equation in slope-intercept form.

**Problem 4.** Find $dy/dx$ for the equation $y - xe^y = 2 - e^x$.

**Problem 5.** Find the limit or prove that it does not exist.

$$\lim_{x \to -\infty} \left( x + \sqrt{x^2 - 5x} \right)$$

**Problem 6.** Find the limit or prove that it does not exist.

$$\lim_{x \to 5} \frac{|x - 5|}{x^2 - 25}$$

**Problem 7.** Use the definition of continuity to determine whether $f$ is continuous at $x = 3$.

$$f(x) = \begin{cases} \dfrac{x^2 - 4x + 3}{x - 3} & \text{if } x \neq 3 \\ 2 & \text{if } x = 3 \end{cases}$$

**Problem 8.** Evaluate the integral.

$$\int \sin^2(4x)\, dx$$

**Problem 9.** Evaluate the integral.

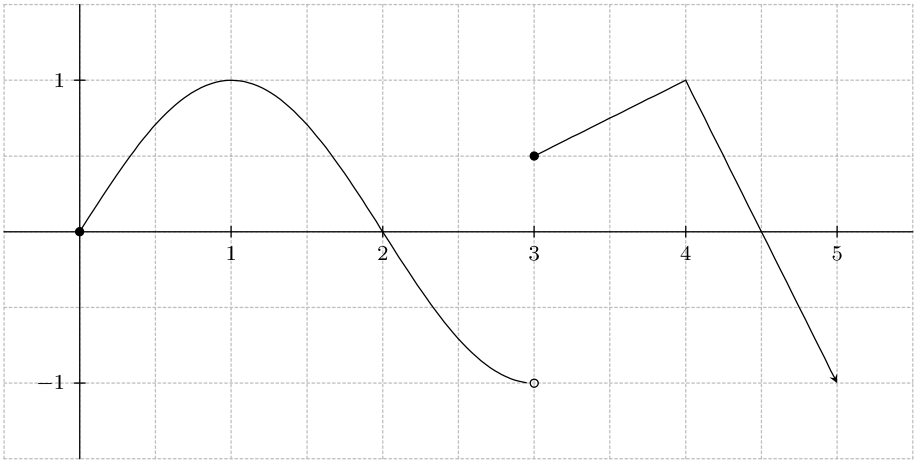
$$\int_0^{\ln 4} \frac{e^x}{3+2e^x}\,dx$$

**Problem 10.** Find the exact area under the graph of $f(x) = x^2$ over the interval $[1, 2]$ using sigma notation and limits of Riemann sums.

**Problem 11.** Evaluate the integral.

$$\int x^3 \sqrt{16 - x^4}\, dx$$

**Problem 12.** The graph of a function $f$ whose domain is $[0, \infty)$ is given below.




(a) Find the critical values of $f$.
(b) Find the intervals where $f'(x)$ is positive.
(c) Arrange the following values in order from smallest to largest: $f'(0.5)$, $f'(1.5)$, and $f'(3.5)$.

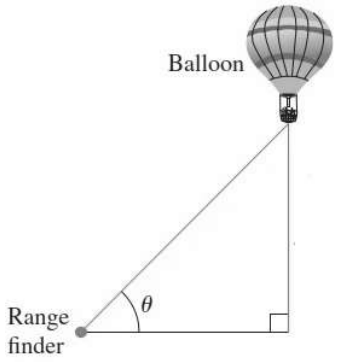
**Problem 13.** Consider the function $f(x) = x^{8/3} - 8x^{2/3}$.

(a) Find the intervals on which $f$ is increasing and decreasing.

(b) Find the local maximum and minimum values of $f$.

(c) The second derivative of $f$ satisfies the following: $f''(x) > 0$ on $(-\infty, 0)$, $f''(x) > 0$ on $(0, \infty)$, and $f''(0)$ does not exist. Use this information, and the information from parts (a) and (b), to sketch a graph of $f$.

**Problem 14.** An observer is 500 feet away from the landing spot of a hot air balloon. The hot air balloon is descending straight down at a rate of 15 ft/sec. Find the rate at which the angle of elevation between the observer and the balloon is changing, when the balloon is 200 feet above the ground.




**Problem 15.** Find the absolute extreme values of $f(x) = 2\cos x + \cos 2x$ on the interval $[0, \pi]$.

**Problem 16.** A particle is moving horizontally along a line. Its position at time $t$ is described by $s(t) = 2t^3 - 21t^2 + 60t$, where $0 \leq t \leq 6$. Find the intervals of time when the speed of the particle is increasing.